Reply to Sergiu I. Vacaru's "Critical remarks on Finsler modifications of gravity and cosmology by Zhe Chang and Xin Li"

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This is our reply to "Critical remarks on Finslerian modifications of gravity and cosmology by Zhe Chang and Xin Li", Sergiu I. Vacaru, Phys. Lett. B 690 (2010) 224. It is pointed out that the Finslerian modifications of gravity and cosmology (Zhe Chang and Xin Li, Phys. Lett. B 676 (2009) 173; *ibid* 668 (2008) 453) is a suggestion on the generalization of Einstein's gravity and cosmology, but not a proof for theorems in geometry. False or true of the theory should be tested by experiments or observations. We show that the arguments of Sergiu I. Vacaru were based a wrong logic. A personal claim can not be used to prove any other theory be wrong. To get the claim: *"we may construct more "standard" physical Finsler classical/quantum gravity theories for metric compatible connections like the Cartan d-connection"* , Sergiu I. Vacaru should complete a consistent presentation at least. We suggest Sergiu I. Vacaru to make some predictions on gravity and cosmoligy using his *"standard" physical Finsler classical/quantum gravity theories* as we did, and compare them with astronomical observations. By the way, we should say that it is still really far from a theory of quantum gravity.

Recently, Sergiu I. Vacaru published critical remarks[1] on our work *Finslerian modifications of gravity and cosmology* [2, 3]. First of all, we thank Vacaru for paying attention to our researches. We are happy to read any kind of criticisms and comments on the papers. The Finslerian modifications of gravity and cosmology are really not complete and in the course of development. However, we found that the comments of Sergiu I. Vacaru was based a wrong logic. The Finslerian modifications of gravity and cosmology is a suggestion on the generalization of Einstein's gravity and cosmology, but not a proof for theorems in geometry. False or true of the theory should be tested by experiments or observations. A personal claim can not be used to prove any other researches to be wrong. To get the claim: *"we may construct more "standard" physical Finsler classical/quantum gravity theories for metric compatible connections like the Cartan d-connection"* , Sergiu I. Vacaru should complete a consistent presentation at least. We suggest Sergiu I. Vacaru to make some predictions on gravity and cosmoligy using his *"standard" physical Finsler classical/quantum gravity theories* as we did, and compare them with astronomical observations.

In the following, we will reply to some key points in the comments of Sergiu I. Vacaru.

Question1: *"The Chern connection is not metric compatible and not the unique connection in Finsler geometry".*

Reply: It is correct. The Chern connection is not metric compatible. The statement we gave in [3] should be replaced by a more clear presentation. In fact, in the second paper of the series[2], just before the formula (2), we have already altered the statement as: "In Finsler manifold, there exists a unique linear connection-the Chern connection. It is torsion freeness and almost metric compatibility". Here the word "unique" just means that the Chern connection is determined by the conditions(or structural equation) of torsion freeness and almost metric compatibility. It should not be read as that the Chern connection is the unique connection in Finsler geometry.

Question2: *"The metric incompatibility make more difficult the definition of spinors and conservation laws in Finsler gravity and does not allow "simple" (super) string and noncommutative generalizations like we proposed."*

Reply: Our paper just presented a classical modification of gravity and cosmology and did not concern any aspect of quantum theory of gravity. Therefore, we do not think it is a comment on our papers. It is strange that a claim on quantum gravity can be used to criticize a classical theory of gravity. We should say that here the logic of Sergiu I. Vacaru is wrong. Even though, we still would like to point out that the comment made a strong conclusion without any proof. It is a pity that a proof of the no go theorem can not be found in the comments[1].

Question3: *"The Ricci tensor introduced by H. Akbar-Zadeh [4] is not correct for all Finsler geometry/gravity models. There were considered various types of Ricci type tensors in Finsler geometries."*

Reply: We do not know any mathematician has presented the theorem. Sergiu I. Vacaru did not give any sound proof about his assertion either. In fact, these var-

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ious types of Ricci tensors in Finsler geometry and the so-called "gravitational field equations" constructed by them depend on the chosen connection. It implies that different gravitational field equations could be obtained while one uses different connections to calculate it, even all the connections are metric compatible. This brings up the problem that which gravitational field equation is the physical one. On the contrary, the Ricci tensor that introduced by Akbar-Zadeh [4] does not face such a problem. It is given as

$$Ric_{\mu\nu} = \frac{\partial^2 \left(\frac{1}{2}F^2 Ric\right)}{\partial y^\mu \partial y^\nu}, \qquad (1)$$

where the Finsler metric is defined as $g_{\mu\nu} \equiv \frac{\partial^2\left(\frac{1}{2}F^2\right)}{\partial y^\mu \partial y^\nu}$, and the Ricci scalar "$Ric$" is the trace of the predecessor of the flag curvature. The flag curvature [5] in Finsler geometry is the counterpart of the sectional curvature in Riemannian geometry. It is a geometrical invariant. Furthermore, the same flag curvature is obtained for any connection that chosen in Finsler space. Thus, the same Ricci tensor is obtained for any connection that chosen in Finsler space Therefore, the Ricci tensor introduced by Akbar-Zadeh [4] is a reasonable and well-defined one.

Question4: *"Sergiu I. Vacaru claimed that he may construct more "standard" physical Finsler classical/quantum gravity theories for metric compatible connections like the Cartan d-connection".*

Reply: To become a theory, a claim should be complete and consistent at least. The most important thing for a physical theory is that it makes predictions that can be tested through experiments and observations. To our point of view, Sergiu I. Vacaru's Finsler gravity theories are still premature. Newton's theory of gravity can not be used to kill Einstein's general relativity. Einstein's equations of gravitational field can not be used to kill the hypothesis of dark energy and dark matter. Another reason for the prematureness is that the Einstein's tensor $E(\hat{D})$ in Sergiu I. Vacaru's Finsler gravity theories is not a conserved quantity [6] (in the sense of covariant differentiation). This is also pointed out by Sergiu I. Vacaru himself in his comments (see the footnote 3 and the formula (3) in [1]). It is well-known that in general relativity the Einstein tensor is a conserved quantity. The Einstein's gravitational field equation constructed in such a form due to the requirement that the energy-momentum tensor must conserve. And this conservation law of energy-momentum tensor is of vital importance and has been extensively used and embedded in different branches of modern physics. Any theory that does not subject to this rule can not be recognized as a physical one. Neither can Sergiu I. Vacaru's Finsler gravity theories. At least, Sergiu I. Vacaru should give a conserved quantity which is the counterpart of energy-momentum tensor in the framework of Finsler geometry. Sergiu I. Vacaru has claimed that the quantum gravity theory is "almost sure" of generalized Finsler type. We wish him publish his claim in an isolated paper. To discuss details on this topic is out of range of our reply.

Acknowledgements

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[1] S. Vacaru, Phys. Lett. B **690** (2010) 224, arXiv:1003.0044v2 [gr-qc].
[2] Z. Chang and X. Li, Phys. Lett. B **676** (2009) 173.
[3] Z. Chang and X. Li, Phys. Lett. B **668** (2008) 453.
[4] H. Akbar-Zadeh, Acad. Roy. Belg. Bull. Cl. Sci. (5) **74** (1988) 281.
[5] D. Bao, S. S. Chern and Z. Shen, *An Introduction to Riemann–Finsler Geometry*, Graduate Texts in Mathematics **200**, Springer, New York, 2000.
[6] S. Vacaru, P. Stavrinos, E. Gaburov, D. Gonta, arXiv:gr-qc/ 0508023.